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DEVELOPMENT OF A LASER VELOCIMETER SYSTEM

J. I. Shipp, R. H. Hines, and W. A. Dunnill

ARO, Inc.

October 1967

ARNOLD ENGINEERING DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
ARNOLD AIR FORCE STATION, TENNESSEE

NOTICES

DEVELOPMENT OF A LASER VELOCIMETER SYSTEM

J. I. Shipp, R. H. Hines, and W. A. Dunnill
ARO, Inc.

FOREWORD

The work reported herein was sponsored by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Program Element 6540223F.

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ABSTRACT

The development of a laser velocimeter for point velocity measurements in liquid and gas flows is discussed. The principle of operation of the velocimeter is based on the Doppler effect, and the velocity measurements are accomplished by an optical homodyning technique. Velocity profiles are presented for laminar flow of water and air in circular flow channels. It was found that a correction factor was required when measuring velocities of fluids having indices of refraction different from air. The velocimeter is shown to be capable of velocity measurements of a vibrating surface.

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NOMENCLATURE

A	Magnitude of the incident beam wave function
Å	Angstrom
B	Magnitude of the scattered beam wave function
c	Velocity of light
D	Diameter of flow channel
$\bar{F}$	Poynting flux
f_D	Doppler frequency, Hz
i	$(-1)^{1/2}$
k_D	$k_s - k_o$
k_o	Magnitude of $\bar{k}_o$
$\bar{k}_o$	Wave vector of the incident laser radiation in S
$\bar{k}_o'$	Wave vector of the incident laser radiation in S′
$k_{o\mu}$	Four-component wave vector of the incident laser radiation in S
$k'_{o\mu}$	Four-component wave vector of the incident laser radiation in S′
k_s	Magnitude of $\bar{k}_s$
$\bar{k}_s$	Wave vector of the scattered laser radiation in S
$\bar{k}_s'$	Wave vector of the scattered laser radiation in S′
$k_{s\mu}$	Four-component wave vector of the scattered laser radiation in S
$k'_{s\mu}$	Four-component wave vector of the scattered laser radiation in S′
n	Index of refraction
R	Radius of the flow channel

r	Distance from centerline in the flow channel
S	Fixed laboratory reference frame
S′	Reference frame moving with velocity $\bar{V}$ relative to S
t	Time
V	Magnitude of $\bar{V}$
$\bar{V}$	Velocity in S
$\bar{V}'$	Velocity in S′
V_m	Maximum velocity in S
$V\mu$	Four-vector velocity in S
$V\mu'$	Four-vector velocity in S
$\langle V \rangle$	Average velocity in S
$\mathring{W}$	Volumetric flow rate
X	Distance along flow channel subtended by θ
x	Magnitude of wave position vector
y	Contracted flow channel diameter
α	Angle between unscattered and scattered light in Fig. 11
β	Angle between the velocity vector and the scattered light in Fig. 3
γ	$\left[1-\left(\frac{V}{c}\right)^2\right]^{1/2}$
Δ	Area
θ	Angle between the incident and scattered light in Figs. 2 and 11
ψ	Angle between the velocity vector and the incident light in Fig. 2
Φ_T	Incident beam wave function
Φ_S	Scattered beam wave function
λ_o	Wavelength of emitted laser radiation
ω_D	Doppler frequency, radians/sec
ω_g	Generator frequency driving the speaker
ω_o	Frequency of incident laser radiation in S
ω_o'	Frequency of incident laser radiation in S′
ω_s	Frequency of scattered laser radiation in S
ω_s'	Frequency of scattered laser radiation in S′

SECTION I
INTRODUCTION

The first laser velocimeter (LV) constructed by Yeh and Cummings (Ref. 1) at Columbia University was used to measure velocities of the order of 10^{-4} m/sec. Later Foreman, George, and Lewis (Ref. 2) constructed an LV which was capable of measuring gas velocities from 10^{-2} to 10^{2} m/sec.

The original purpose of developing the laser velocimeter at the AEDC was to study flow fields of low speed V/STOL aircraft. Since the developmental plan was established, however, interest has arisen also in the high speed regime encountered in shock tubes and in jet plume studies, and at present, theoretical effort is being focused on these problems. Simultaneously, experiments have been initiated in measuring very low speed boundary-layer velocities in electrically conducting fluids of MHD flow channels. Because of the wide range of possible applications of this instrument, the objective of this program is to define the repeatability and resolution of the instrument and to extend the range from a few tenths of a centimeter per second to about 10^{4} m/sec.

The use of the LV for point velocity measurements has many advantages over conventional measuring techniques. The flow field is not measurably perturbed by the LV, since only a minute amount of the low power laser beam is absorbed by the fluid. The size of the focal point of a focused laser beam can be as small as a few microns indicating that the spatial resolution of the LV is typically of the order of a few microns. Also, the response of the LV is linear over the entire velocity range of interest in normal fluid flow studies. The range of velocities capable of being measured by the laser technique is limited by the frequency response of the readout instrumentation. Velocity measurements from a small fraction of a centimeter per second to 10^{3} m/sec are currently possible. The extension of the upper range by two orders of magnitude appears theoretically possible. The LV has the added capability of measuring rapidly varying velocities which are experienced in fully developed turbulence at Reynolds numbers greater than 10^{5} (Ref. 3).

SECTION II
THEORY OF OPERATION

The principle of operation of the laser velocimeter is based on the Doppler effect. A schematic diagram of the LV system is shown in Fig. 1 (Appendix). Light emitted from the laser is focused to a point in the flowing

fluid by means of lens L_1. From this point, a small portion of the beam is scattered through the angle θ. This scattered light is shifted in frequency because of the Doppler effect and is focused by lens L_2. The frequency shifted light is then incident upon a front surface mirror M_1, reflected through the beam splitter, and focused on the photocathode surface. The unscattered portion of the laser beam is focused by lens L_3, attenuated by a neutral density filter, transmitted through the beam splitter, reflected at the front surface mirror M_2, and rotated 90 deg by the beam splitter so that it becomes incident on the photocathode surface. The paths of both beams between the beam splitter and the photocathode must be coincidental. At the photocathode surface, the two beams are optically homodyned to obtain the Doppler frequency. Hence, the frequency detected by the photomultiplier is proportional to the velocity at the scattering point.

2.1 DERIVATION OF THE DOPPLER EQUATION

Consider a scattering center fixed in a reference frame S′ which has a velocity $\bar{V}$ relative to a laboratory fixed frame of reference S. A light source of frequency ω_o and wave vector $\bar{k}_o$ in S will appear as ω_o' and $\bar{k}_o'$ in S′. The incident light is scattered by the scattering center in S′. The angular frequency of the scattered light is identical to the incident light as viewed in S′, i.e.

$$\omega_o' \equiv \omega_s' \tag{1}$$

To obtain the Doppler shift frequency, ω_D, the frequency of the scattered light must be calculated in the laboratory system, S, where ω_D is defined as

$$\omega_D' \equiv \omega_s - \omega_o \tag{2}$$

Use is made of the invariance of scaler products of four vectors, where the following quantities transform as four vectors:

$$\begin{aligned}
k_{o\mu} &= (\bar{k}_o, i\ \frac{\omega_o}{c}) \\
k'_{o\mu} &= (\bar{k}_o', i\ \frac{\omega_o'}{c}) \\
k_{s\mu} &= (\bar{k}_s, i\ \frac{\omega_s}{c}) \\
k'_{s\mu} &= (\bar{k}_s', i\ \frac{\omega_s'}{c}) \\
V_\mu &= (\gamma\bar{V}, i\gamma c) \\
V_\mu' &= (\gamma\bar{V}', i\gamma c) = (0, i\gamma c)
\end{aligned} \tag{3}$$

where

$$i = (-1)^{1/2} \qquad \mu = 1, 2, 3, 4$$

$$c = \text{velocity of light}$$

and

$$\gamma = [1 - (\frac{V}{c})^2]^{1/2}$$

Since four-vector scaler products are invariant,

$$k_{o\mu} V_{\mu} = k'_{o\mu} V'_{\mu} \tag{4}$$

and

$$k_{s\mu} V_{\mu} = k'_{s\mu} V'_{\mu} \tag{5}$$

where the Einstein summation convention has been employed. Substitution of the definitions in Eq. (3) into Eqs. (4) and (5) yields

$$k_{o\mu} V_{\mu} = -\omega'_o \gamma \tag{6}$$

and

$$k_{s\mu} V_{\mu} = -\omega'_s \gamma \tag{7}$$

Hence,

$$k_{o\mu} V_{\mu} = k_{s\mu} V_{\mu} \tag{8}$$

In three-vector form, Eq. (8) becomes

$$\gamma \bar{k}_o \cdot \bar{V} - \gamma\omega_o = \gamma \bar{k}_s \cdot \bar{V} - \gamma\omega_s \tag{9}$$

Equation (2) then leads to

$$\omega_D = (\bar{k}_s - \bar{k}_o) \cdot \bar{V} \tag{10}$$

Since

$$|\bar{k}_s| \approx |\bar{k}_o| \equiv \frac{2\pi n}{\lambda_o}$$

and

$$f_D = \frac{\omega_D}{2\pi} \tag{11}$$

Equation (10) becomes

$$f_D = \frac{2nV}{\lambda_o} \sin\left(\frac{\theta}{2}\right) \sin\left(\psi + \frac{\theta}{2}\right) \tag{12}$$

where n is the index of refraction of the medium, λ_o is the wavelength of the emitted radiation, and θ and ψ are the angles defined in Fig. 2.

2.2 DERIVATION OF THE OPTICAL HOMODYNE EQUATION

In this section, a simplified derivation of the optical homodyne process is derived. The scattered and the transmitted beams are assumed to be aligned so that they leave the beam splitter at the same

point and are incident upon the same point of the photocathode. The wave functions for the transmitted and scattered beams are approximated by plane waves of the form

$$\Phi_T = A\, e^{ik_o x - i\omega_o t} \tag{13}$$

$$\Phi_S = B\, e^{i(k_o + k_D)x - i(\omega_o + \omega_D)t} \tag{14}$$

By the superposition theorem, the two wave functions add linearly. Hence,

$$\begin{aligned}\Phi = \Phi_T + \Phi_S &= A\, e^{ik_o x - i\omega_o t} + A\, e^{i(k_o + k_D)x - i(\omega_o + \omega_D)t} \\ &\quad + (B - A)\, e^{i(k_o + k_D)x - i(\omega_o + \omega_D)t}\end{aligned} \tag{15}$$

The photocathode current is proportional to the Poynting flux, time averaged over the optical period, which in turn is proportional to the real part of $\Phi^*\Phi$. Multiplication of Eq. (15) by its complex conjugate leads to

$$\bar{F} \approx \mathrm{Re}\,(\Phi * \Phi) = 2A^2 + 2A(B - A) + (B - A)^2 + 2AB \cos (k_D x - \omega_D t) \tag{16}$$

From Eq. (16), it is seen that photocathode tube current is composed of a constant or d-c term and an a-c term that varies at the Doppler angular frequency ω_D. The laser velocimeter measurement thus consists of measuring the frequency of a signal given by Eq. (16) and converting it to a velocity by Eq. (12).

SECTION III
EXPERIMENTAL ARRANGEMENTS

During the course of development of the laser velocimeter, several experimental arrangements were used. First, a mirror was used as the moving scattering source. The mirror was attached to the cone of a loud speaker which was driven by a signal generator in simple harmonic motion. The experiment was designed to investigate optical alignment techniques and phototube response. Later, velocities of fluid media were studied under various flow conditions.

3.1 VIBRATING MIRROR EXPERIMENT

The optical system shown in Fig. 3 includes a scattering source moving in the simple harmonic motion of the speaker cone. Thus, the velocity component in the direction of the scattered light beam varies

sinusoidally. The time dependence of the velocity is given by Eq. (17).

$$V = V_m \cos(\omega_g t) \cos \beta \tag{17}$$

where ω_g is the generator frequency driving the speaker, and β is the angle between the velocity and the scattered beam. From Eqs. (12) and (16), it is seen that the phototube output should be a signal that is frequency modulated at the Doppler frequency at a modulation rate determined by the generator frequency. A typical oscillogram of the phototube output for a speaker frequency of 50 Hz is shown in Fig. 4. The end points of the harmonic oscillation are represented by the low frequency portion of the trace; and the point of maximum velocity, the equilibrium position of the oscillator, is represented by the high frequency portion of the trace. All frequencies corresponding to velocities from zero to $V_m \cos \beta$ are present in the homodyne signal as shown in the spectrogram of the signal in Fig. 5.

Since the results of the vibrating mirror studies indicated that the LV could be used for vibration studies, a diffuse aluminum surface was substituted for the quarter wave mirror. The results were comparable to those obtained in the mirror experiment with the exception that the amplitude of the output signal was lower because of the diffuse scattering from the aluminum surface.

3.2 LIQUID VELOCITY MEASUREMENTS

The LV was used to measure point velocities in water flow which was seeded with half-micron polystyrene latex spheres. The specific gravity of the seed particles is unity; therefore, settling of these particles in the chamber did not occur. A schematic diagram of the liquid flow system is shown in Fig. 6. In this particular series of experiments, the flow was laminar since the Reynolds number was always below 2000.

The optical system was mounted on a moveable table as shown in Fig. 7.

The system was traversed so that the focal point of the focusing lens moved across the flow channel diameter. The position of the optical table, and hence the position of the focal point relative to the tube walls, was determined to within 5×10^{-4} cm by a radial micrometer.

A typical oscillogram of the homodyne signal for laminar flow is shown in Fig. 8. The signal was amplified and filtered to remove high frequency noise. A typical spectorgram of the phototube output is shown in Fig. 9. The Doppler signal was 125 kHz as seen from the upper and

lower sideband frequencies of the spectrum analyzer. Data of this type were obtained at 0.025-cm increments along the tube diameter. A plot of the velocity profile obtained by this method at a point 1.3 m from the entrance area, defined in Fig. 6, is shown in Fig. 10. The solid line is the theoretically predicted parabolic velocity profile (see for example Ref. 4) of a fluid flowing in a smooth pipe in laminar flow as described by

$$\frac{V}{V_m} = 1 - \frac{r}{R}\Big)^2 \tag{18}$$

where R is the tube radius and V_m is the centerline velocity. The volumetric flow rate, $\dot{W}$, is related to the average velocity, $\langle V\rangle$, by

$$\dot{W} = \langle V\rangle\, \Delta \tag{19}$$

where Δ is the area of the tube, and $\langle V\rangle$ is obtained by integrating Eq. (18).

$$\langle V\rangle = V_m \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^{R} \left[1 - \left(\frac{r}{R}\right)^2\right] r\,dr\,d\Phi \tag{20}$$

yielding

$$\langle V\rangle = \frac{V_m}{2} \tag{21}$$

Substituting Eqs. (19) and (21) into (18) gives

$$V = \frac{2\dot{W}}{\Delta}\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{22}$$

For the experimental conditions ψ = 90 deg, θ = 10.3 deg, n = 1.33 and λ_o = 6328 Å, the centerline Doppler frequency was measured to be 24.5 kHz which corresponds to a V_m of 8.68 cm/sec. V_m calculated from Eq. (22) is 8.52 cm/sec. The error of 1.7 percent is well within the accuracy of the flow rate measurement.

When comparing the theoretical velocity profiles with experimentally measured values, a correction factor for the distance traversed is required because of the index of refraction of the fluid media. For fluid media with indices of refraction greater than that of air, the distance traversed by the focal point of the laser beam in the fluid appears contracted when compared to the actual distance that the optics are moved. This is seen by considering the geometry shown in Fig. 11.

The distance y traversed by the optical table is

$$y = \frac{X}{\tan \alpha} \tag{23}$$

where

$$X = \frac{D}{\tan\left(\frac{\pi}{2} - \theta\right)} \tag{24}$$

From Snell's law

$$n = \frac{\sin \alpha}{\sin \theta} \tag{25}$$

Thus the ratio of the table movement distance to the actual traverse distance in the medium, obtained by combining Eqs. (23), (24), and (25), is

$$\frac{y}{D} = \left\{\tan \alpha \tan \left[\frac{\pi}{2} - \sin^{-1}\left(\frac{\sin \alpha}{n}\right)\right]\right\}^{-1} \tag{26}$$

For air flow n is equal to unity, and from Eq. (25) it can be shown that

$$\lim_{n \to 1} \left(\frac{y}{D}\right) = 1 \tag{27}$$

A plot of $\frac{y}{D}$ as a function of the scattering angle, α, (for water flow $n = 1.33$) is shown in Fig. 12. This correction factor was applied to the data in Fig. 10.

3.3 GAS VELOCITY MEASUREMENTS

The LV was used to measure point velocities in smoke seeded air under laminar flow conditions. The flow system used in the experiments is shown in Fig. 13. A typical spectrogram of the homodyne signal is shown in Fig. 14, and a plot of the measured velocity profile and the velocity profile calculated from Eq. (22) is shown in Fig. 15. Again the experimental agreement is within the accuracy of the measured flow rate.

SECTION IV
CONCLUSION

The velocity of a vibrating mirror and point velocity measurements in water and gas laminar flows have been obtained using the LV. The data obtained for the fluid flows were experimentally verified by independent means. The velocity profiles of the two experiments were found to deviate by less than 5 percent from the theoretical curves, which is well within the experimental error involved with measuring the angles θ and ψ and the volumetric flow rate $\dot{W}$. Since the quantitues n and λ_0 in Eq. (12) are accurately known, the accuracy of the LV is determined by the accuracy to which the angles θ and ψ can be measured.

Efforts are underway to measure velocity distributions at points in a fluid in fully developed turbulence. Coupled with this effort, a direct readout system for obtaining real time velocity measurements will also be developed. Plans call for extending the range of the LV velocity measurements to approximately 10^4 m/sec.

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APPENDIX
ILLUSTRATIONS

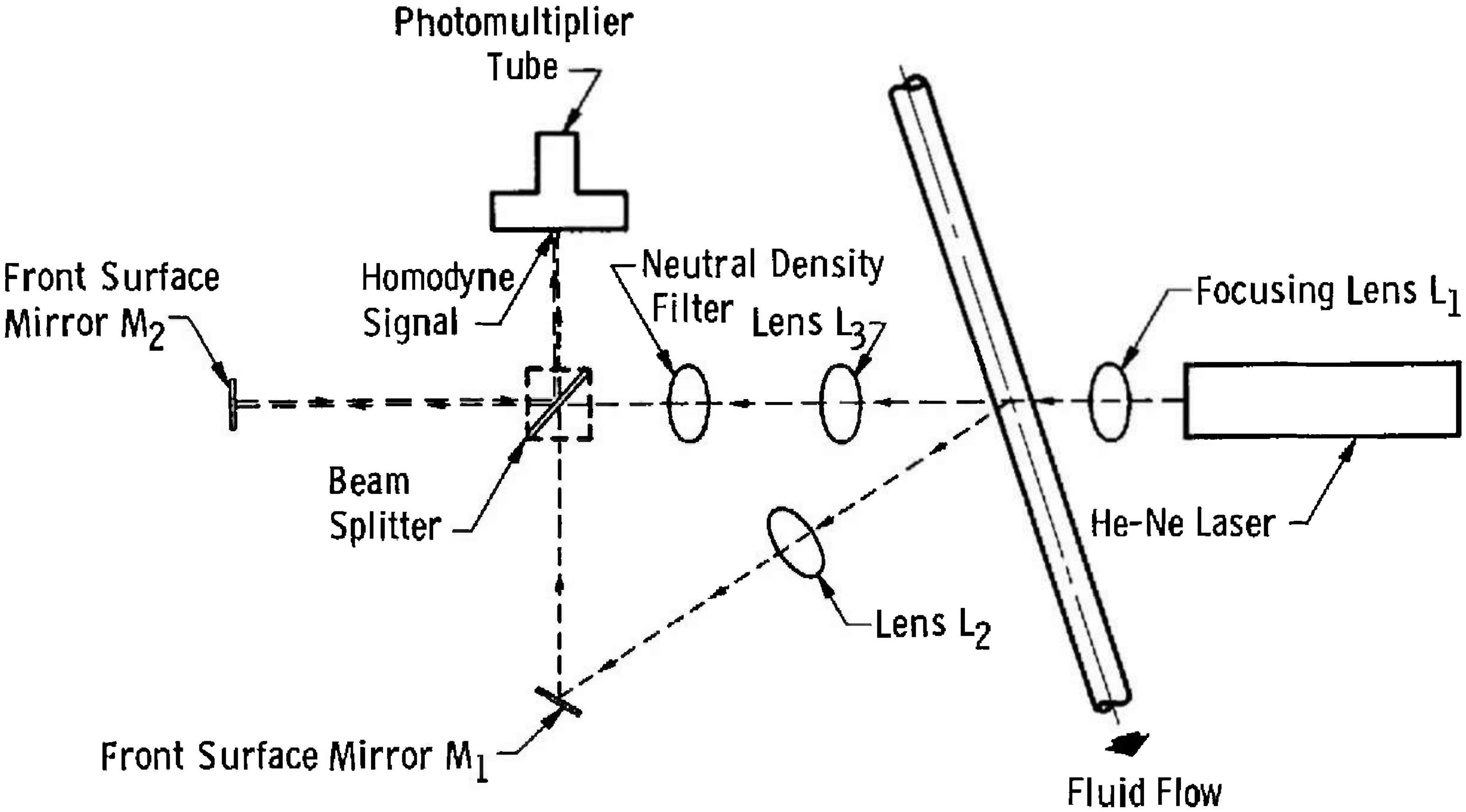


Fig. 1 Schematic Diagram of a Typical Laser Velocimeter System

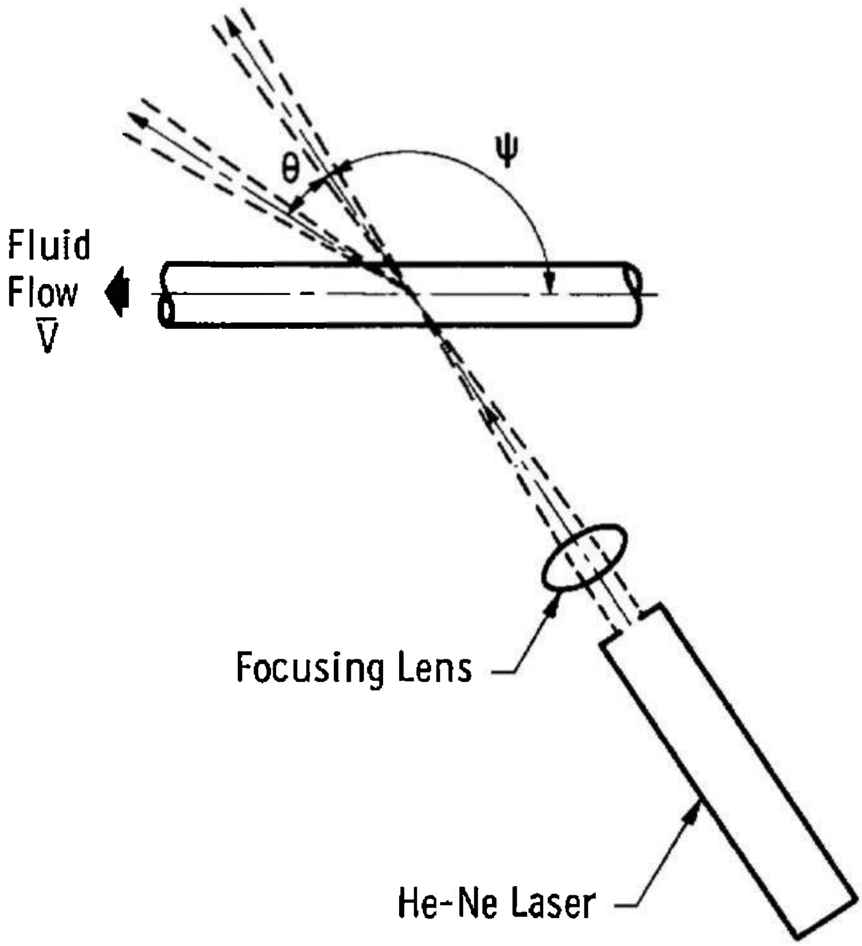


Fig. 2 Angle Definitions for Doppler Effect Calculation

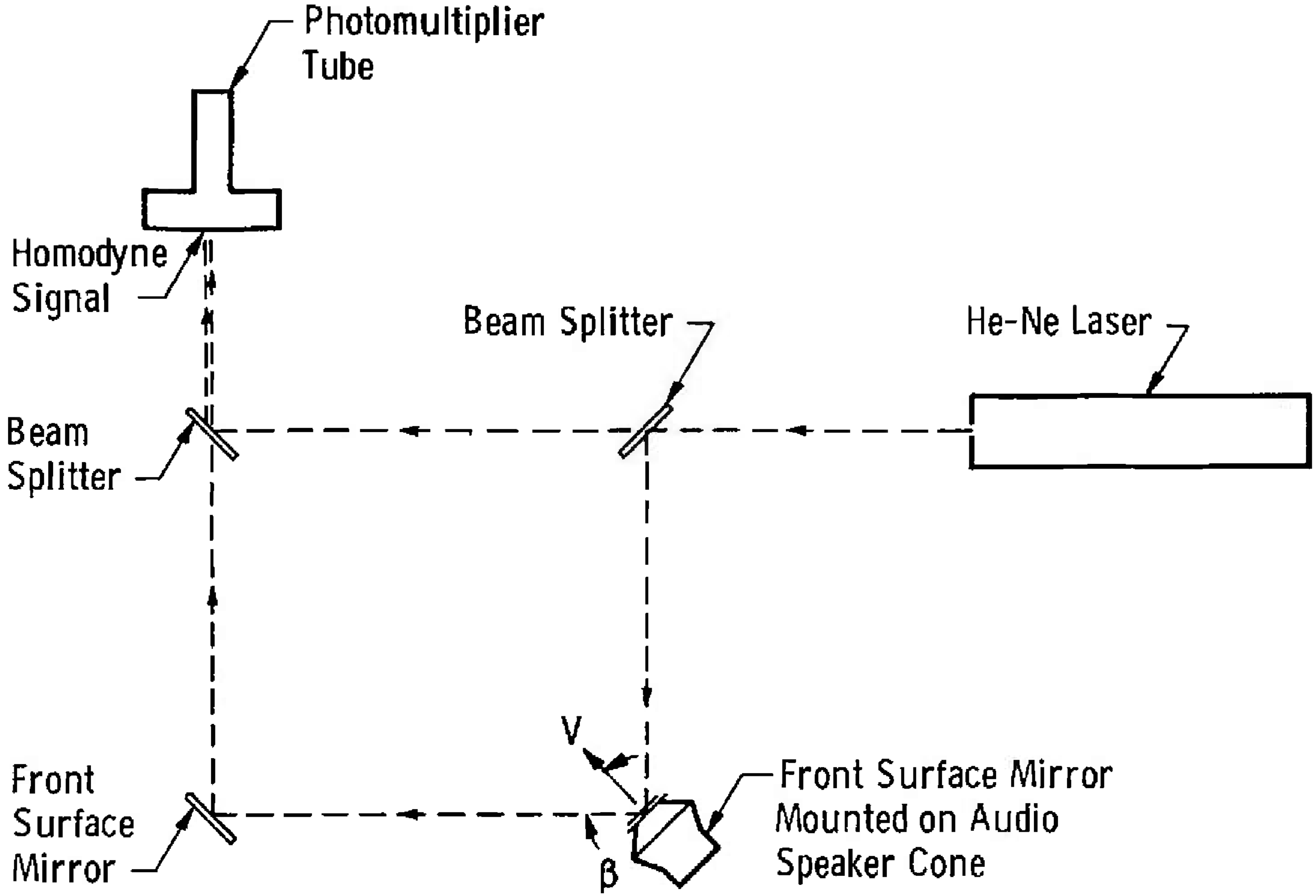
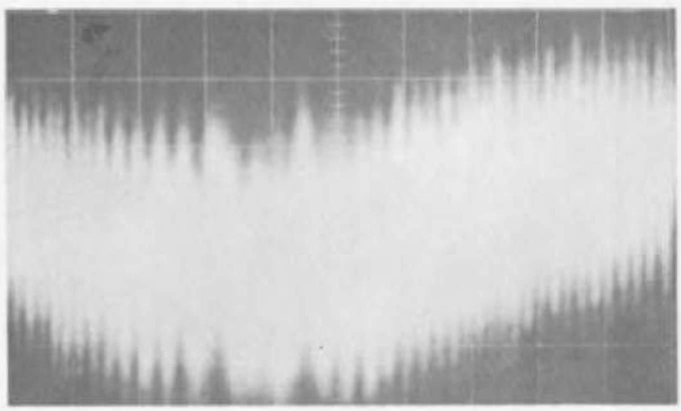


Fig. 3 Optical System for the Vibrating Mirror Experiment

1 v/cm



0.2 μ sec/cm

Fig. 4 Oscillogram of the Vibrating Mirror Signal

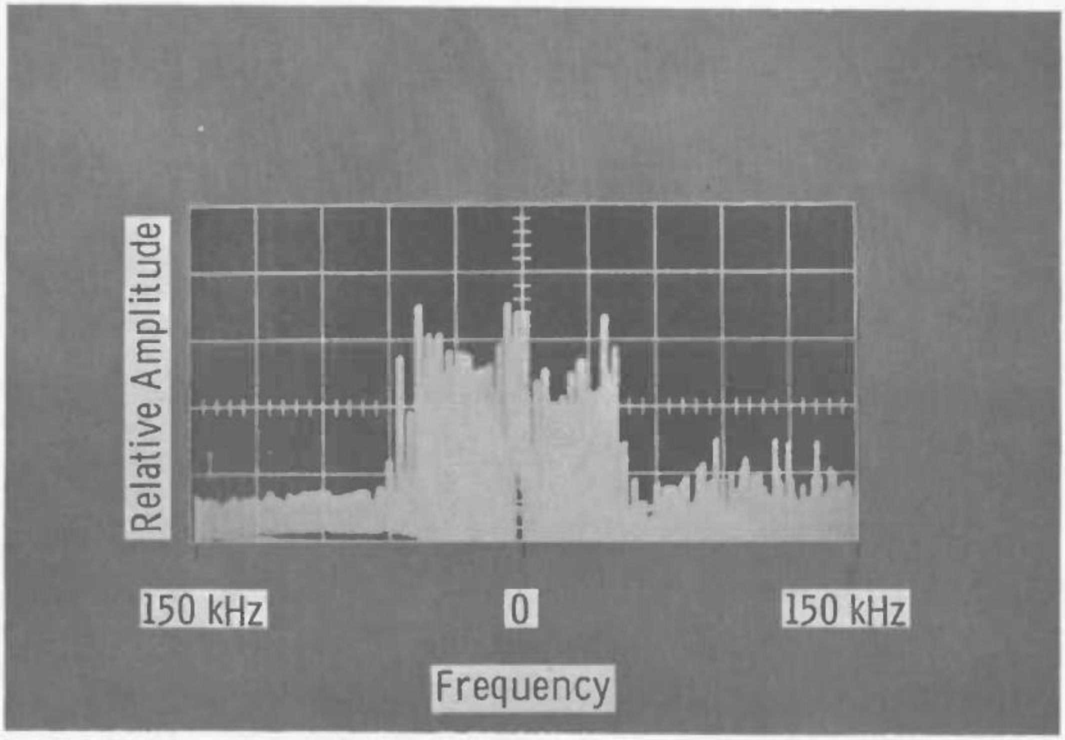


Fig. 5 Spectrogram of the Vibrating Mirror Signal

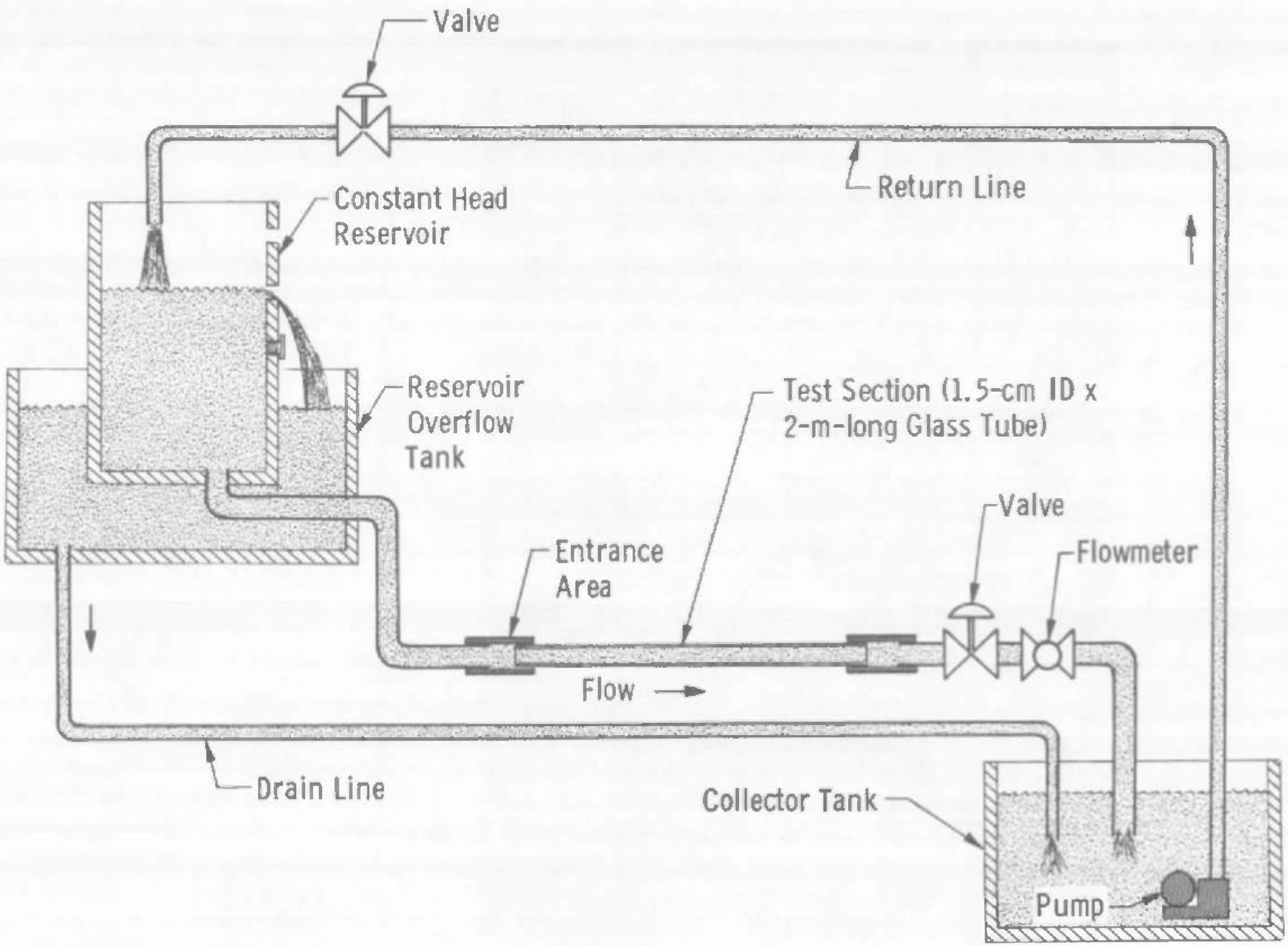


Fig. 6 Schematic Diagram of Liquid Flow System

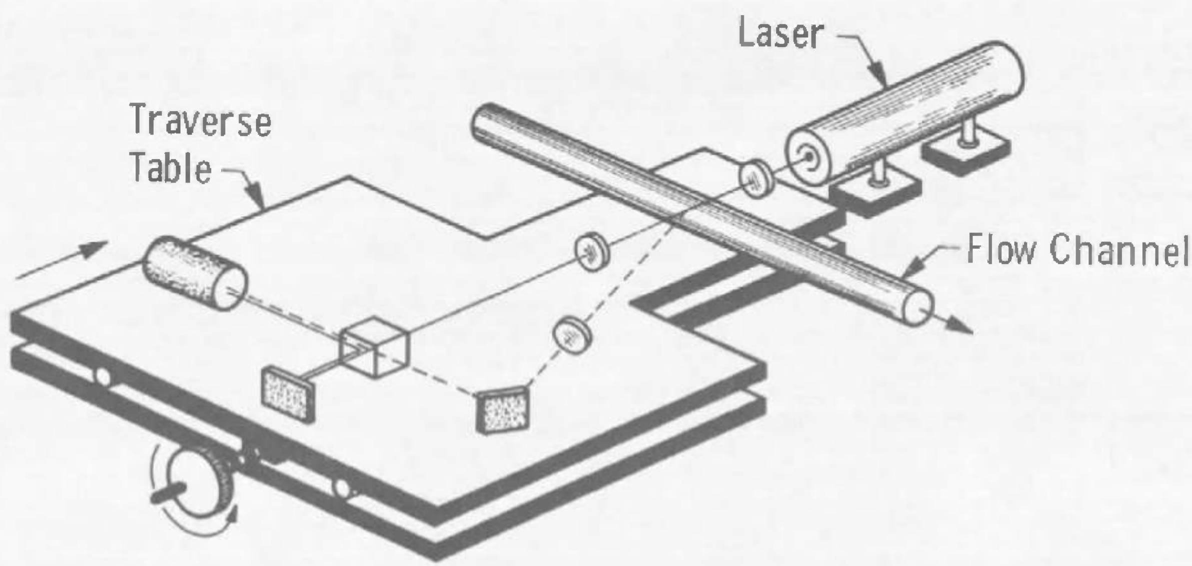


Fig. 7 Traverse Table Arrangement

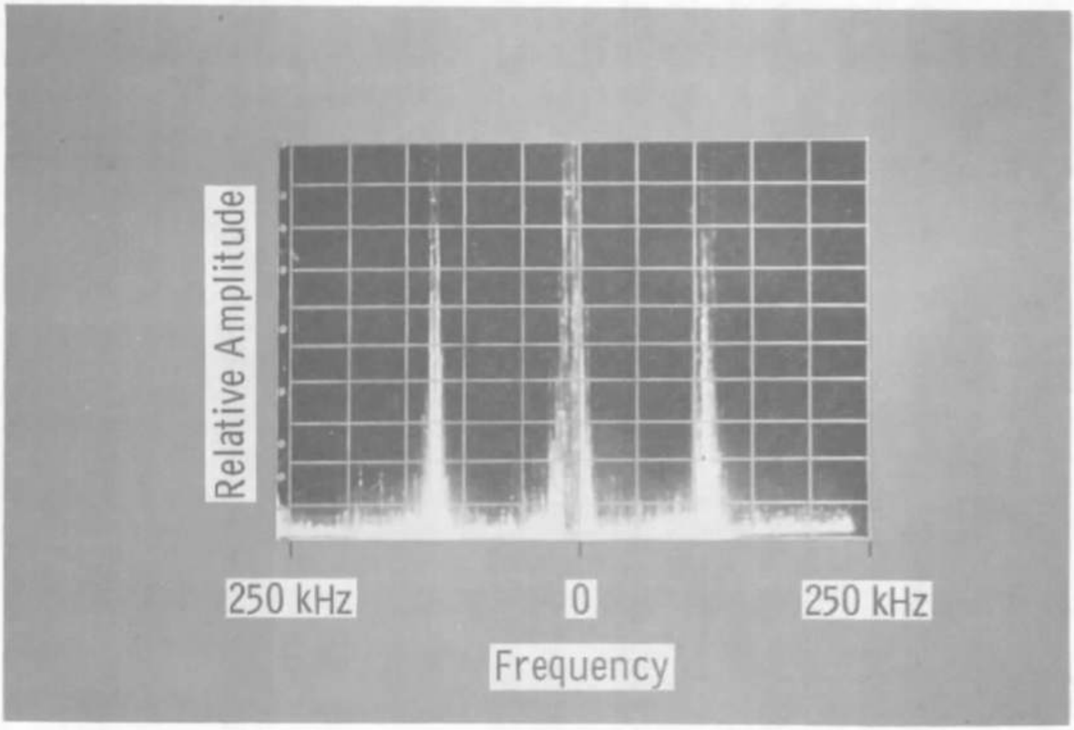
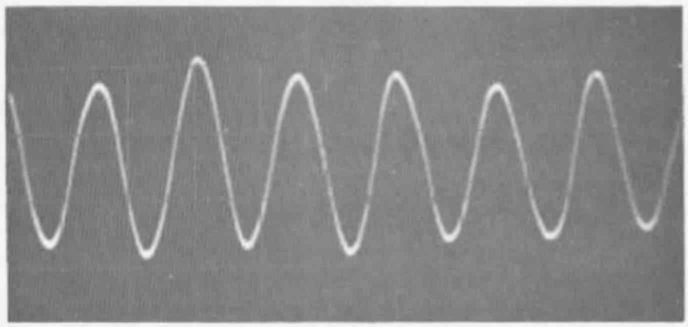


Fig. 8 Typical Spectrogram from the Water Velocity Measurements

20 mv/cm



5 μ sec/cm

Fig. 9 Typicol Photomultiplier Output Signal

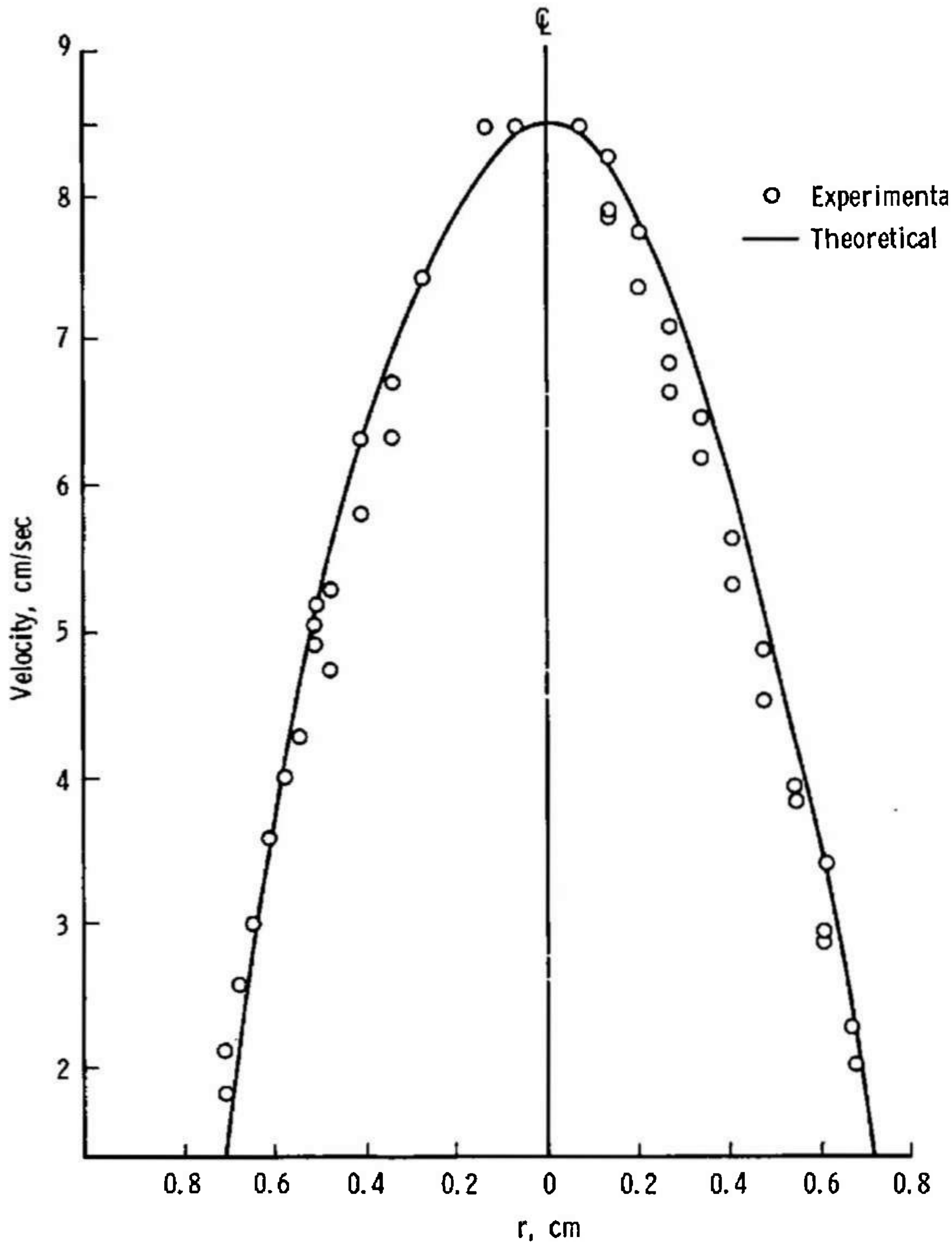


Fig. 10 Velocity Profile of Water in a Circular Pipe

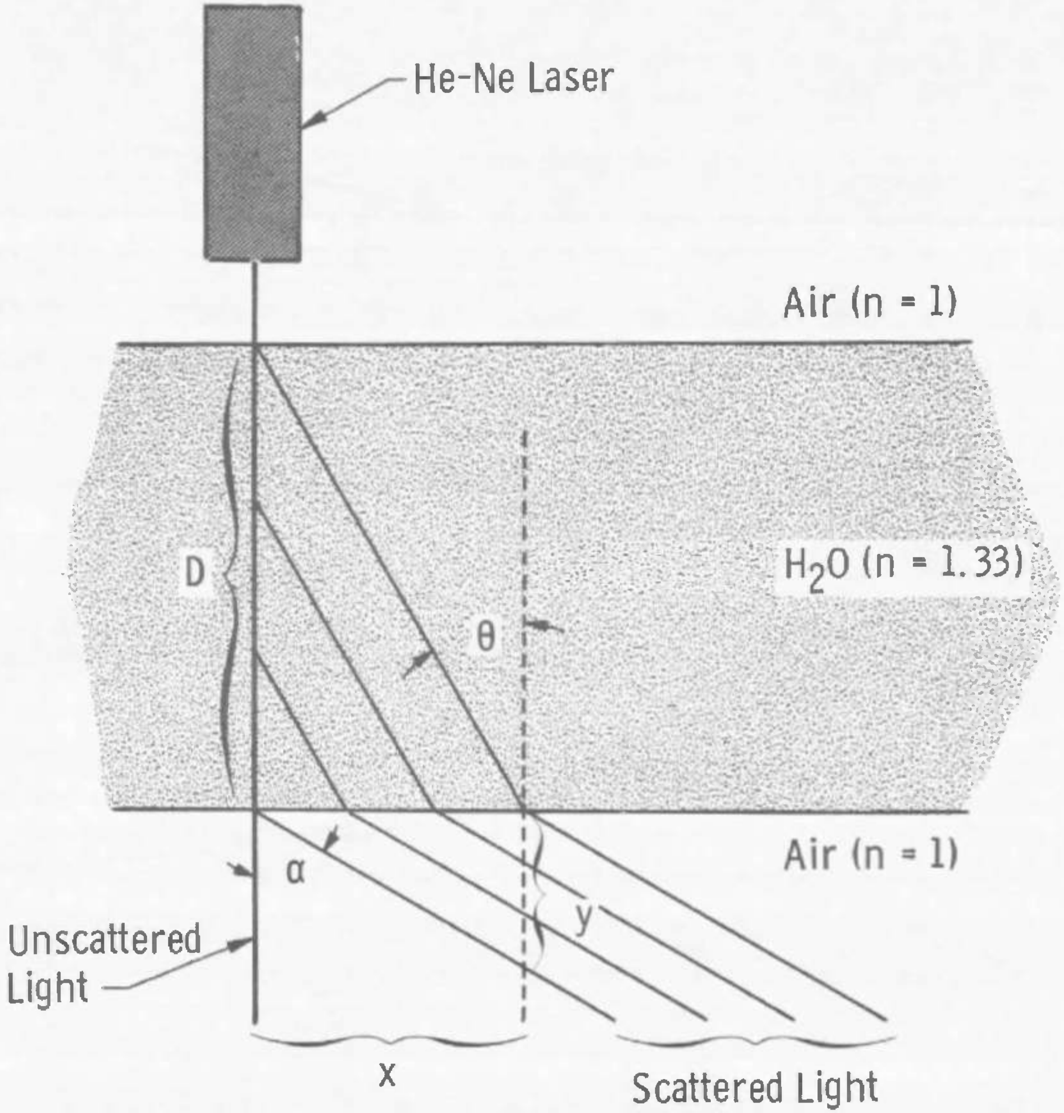


Fig. 11 Geometry for Correction Factor Calculation

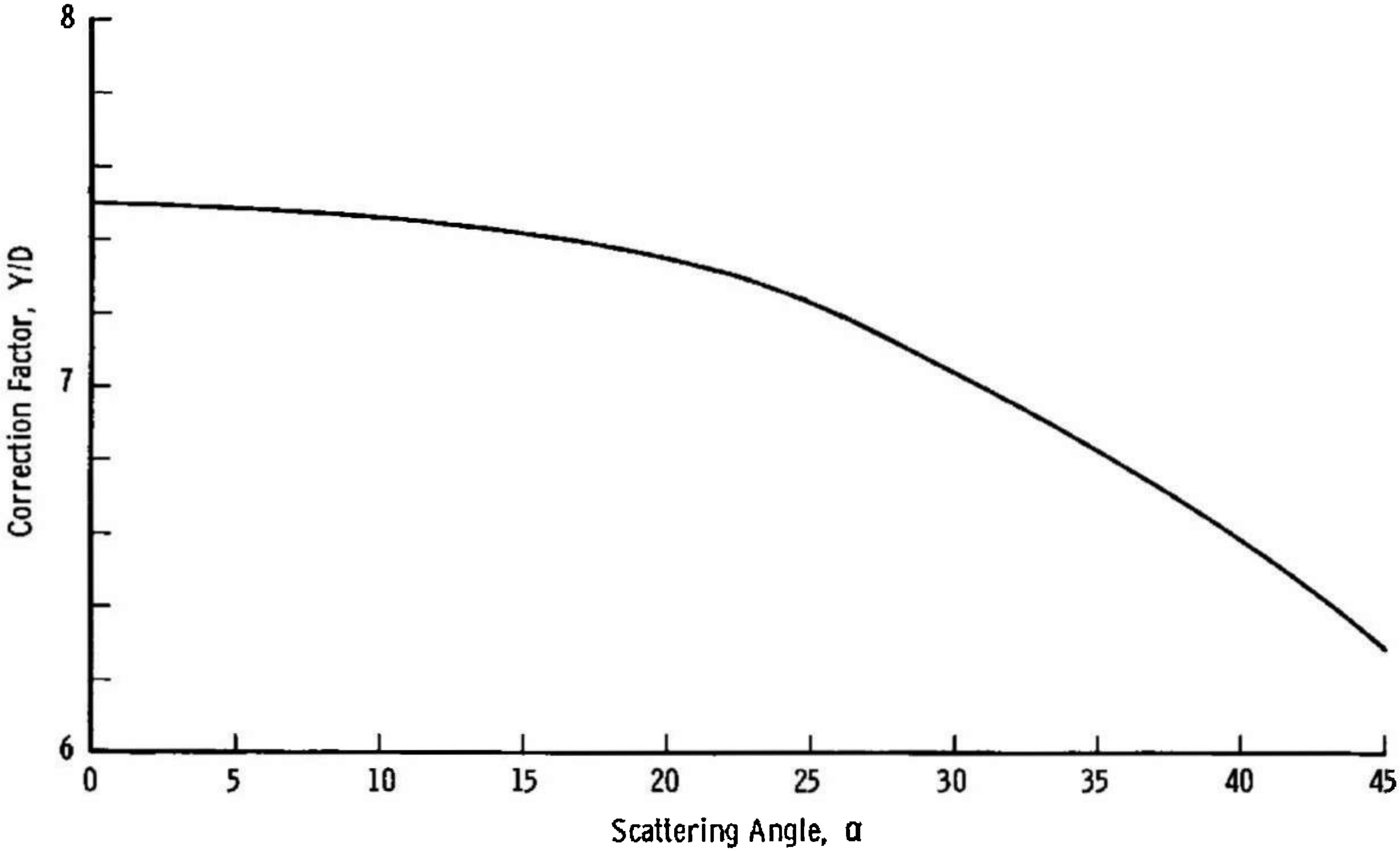


Fig. 12 Correction Factor versus Scattering Angle

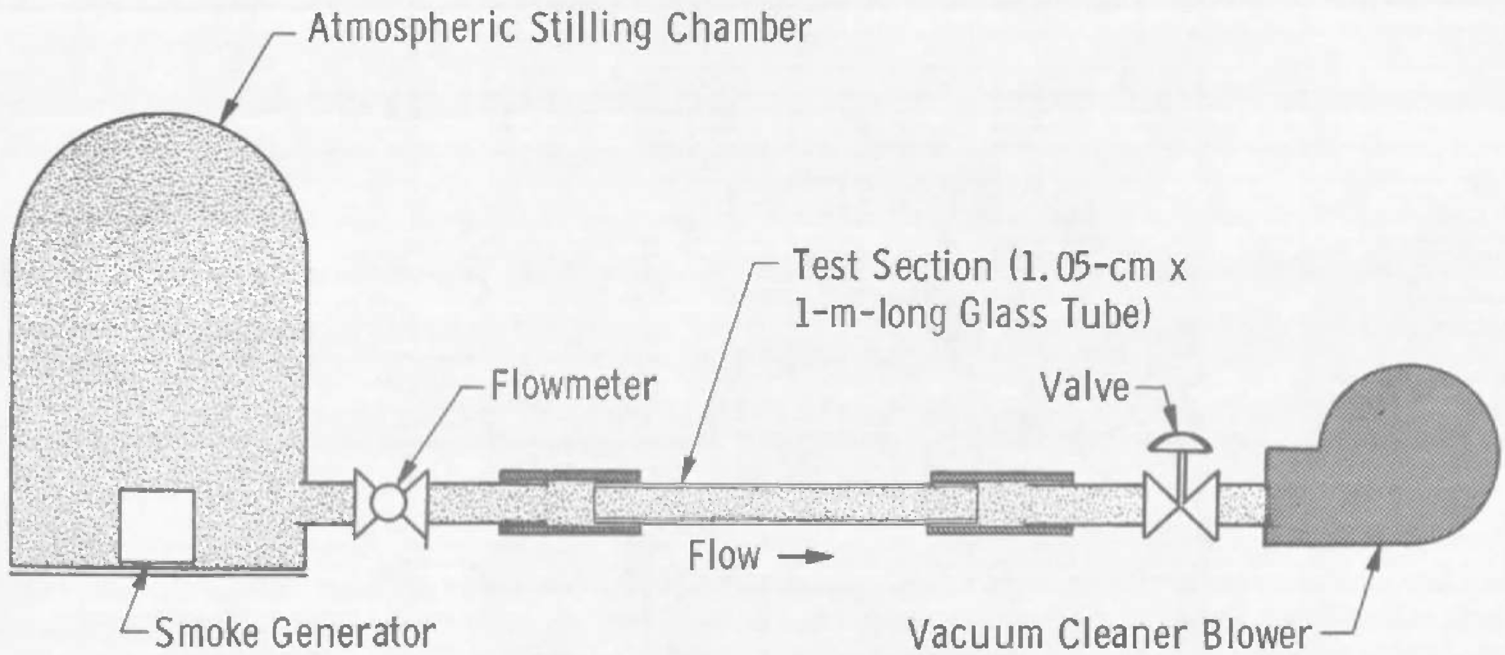


Fig. 13 Flow System for Velocity Measurements in Air

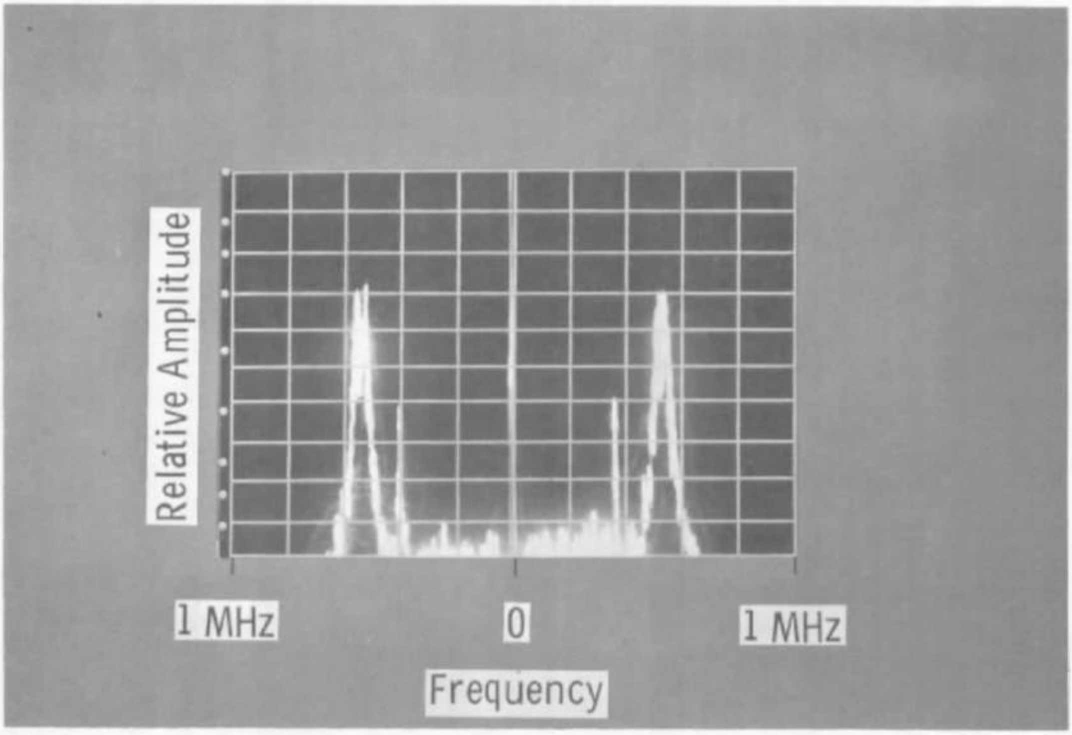


Fig. 14 Typical Spectrogram from the Air Velocity Measurements

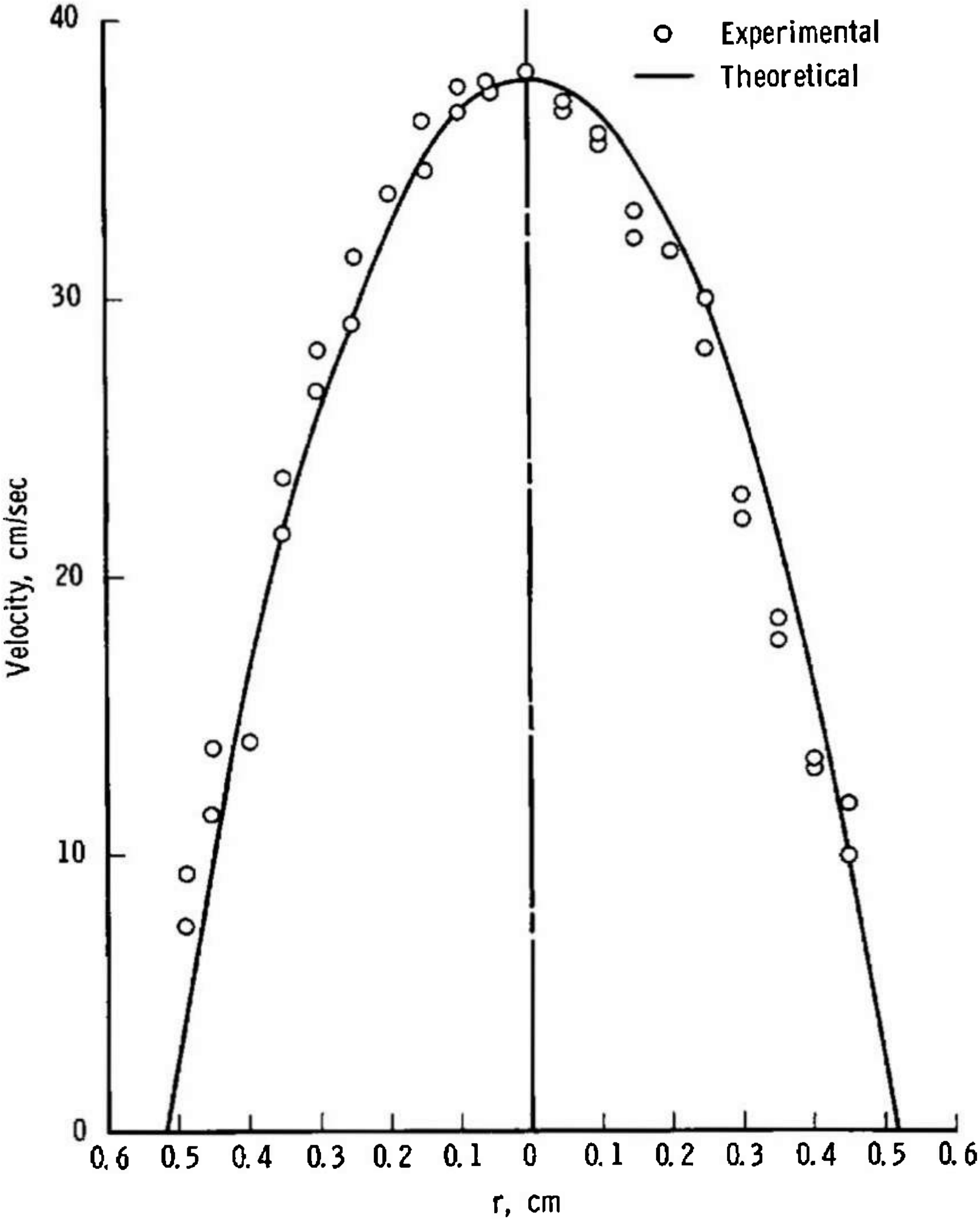


Fig. 15 Velocity Profile of Air in a Circular Pipe

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13 ABSTRACT

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14. KEY WORDS	LINK A		LINK B		LINK C	
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1 laser velocimeters						
gas flow measurement						
V/STOL aircraft						
shock tubes						
jet plumes						

2. velocemeters
3. Liquids .. Velocity
4 gases -- "

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